CLEARING HOUSE, MARGIN REQUIREMENTS, AND SYSTEMIC RISK

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Margins are the major safeguards against default risk on a derivatives exchange. When the clearing house sets margin requirements, it does so by only focusing on individual clearing firm positions (e.g., the SPAN system). We depart from this traditional approach and present an alternative method that accounts for interdependencies among clearing members when setting margins. Our method generalizes the SPAN system by allowing individual margins to increase when clearing firms are more likely to be in financial distress simultaneously.

Recent turmoil in financial markets has heightened the need for well-functioning clearing facilities in derivatives markets, particularly when large market participants are in financial distress and eventually default (Acworth 2009; Pirrong 2009; Duffie and Zhu 2010). In a derivatives exchange, the clearing house is responsible for the clearing function, which consists of confirming, matching, and settling all trades. The clearing house operates with a limited number of clearing firms or futures commission merchants, which are private firms that have the right to clear trades for themselves (i.e., proprietary trading), for their own customers, and for the customers of non-clearing firms.[1]

1. While derivatives clearing systems have been developed to deal with exchange-traded futures and options, there is strong pressure to force over-the-counter derivatives to go through similar clearing processes (Acharya et al. 2009; U.S. Congress' OTC Derivatives Market Act of 2009; U.S. Department of Treasury 2009; Duffie, Li, and Lubke 2010). In response, the CME, Intercontinental Exchange, and EUREX have recently created clearing facilities for Credit Default Swaps (CDS).

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In order to mitigate default risk, the clearing house requires clearing members to post margin (i.e., collateral). At the end of each day, the clearing house marks-to-market all outstanding trading positions and adjusts margins accordingly. A problematic situation arises, however, when the daily loss of a clearing firm exceeds its posted collateral. In this case, the firm may decide to default on its obligations, and the clearing house may have to draw on its default fund to compensate the winning counterparties.[2] Eventually, the clearing house may default as well after its default fund has been exhausted. This scenario, as unlikely as it may appear, is plausible, especially if several large clearing firms are in financial distress and ultimately default. It is also economically significant, because the failure of a clearing house would cause a major systemic shock that could spread default risk throughout the financial system.

Current practice on derivatives exchanges is to set the margin level of a derivative contract in such a way that it leads to a given target probability of a loss in excess of the margin (Figlewski 1984; Booth et al. 1997; Cotter 2001). Similarly, for a portfolio of derivatives, the margin requirement is derived from a distribution of simulated losses associated to the current portfolio positions (e.g., the Chicago Mercantile Exchange's SPAN system). We depart from this traditional view and account for tail dependence in the losses of clearing firms when setting collateral requirements. More specifically, we allow the margin requirements of a particular firm to depend not only on its own trading positions but also, potentially, on other clearing firms' positions. The basic intuition behind this concept is that the collateral requirement for a given clearing firm should increase when it is more likely to experience financial distress at the same time as other clearing firms.

Joint financial distress and defaults are more likely to occur when the trading positions of different clearing firms are similar or when they have similar risk exposures. Conceptually, the main cause of correlated trading across large clearing firms is that they share a common (and superior) information set. This informational advantage leads to similar directional trades. Furthermore, much of the proprietary trading activity on derivatives exchanges consists of arbitraging futures and over-the-counter markets or cash markets (e.g., cash-futures arbitrage of the S&P 500 index and eurodollar-interest rate swap arbitrage). As a result, if large clearing firms exploit similar arbitrage opportunities, they will have similar trading positions. Empirical evidence of correlated trading among large financial institutions is found in many settings, including futures markets. Using data for all Chicago Mercantile Exchange's (CME) clearing firms, for instance, Jones and Pérignon (2010) show that extreme losses by systemically-important clearing firms tend to cluster. This finding suggests that the derivative positions of the largest trading firms can be at times very similar.

Our approach for computing margins can be summarized as follows. We start from the trading positions of each clearing firm at the end of a given day. We then

2. Although exogenous events unrelated to futures losses might also result in default, we do not specifically address these situations.

consider a series of scenarios in which both the level and the volatility of all underlying assets are shocked by an arbitrary amount – in the spirit of *stress testing*. For each scenario, we mark-to-model the clearing firm's portfolio and compute the associated hypothetical profit-and-loss (hereafter P&L). The standard collateral requirement of each clearing firm is equal to the q% quantile of the simulated P&L across all considered scenarios. Then for each pair of clearing firms, we compute the coefficient of lower tail dependence from the vectors of hypothetical P&L of both firms. This coefficient is defined as the probability of two clearing members having simultaneous extreme trading losses. We then set the collateral requirement of each clearing firm as a function of the highest coefficient of tail dependence between this firm and every other clearing firm. We show that accounting for interdependencies among clearing members reduces the likelihood of several clearing members being simultaneously in financial distress, as well as, the magnitude of the margin shortfall given joint financial distress, which greatly lowers systemic risk concerns.

Our methodology displays several attractive features. First, it is perfectly compatible with existing risk management techniques in place in derivatives exchanges, such as the SPAN system (Chicago Mercantile Exchange 2009). Second, our methodology can be applied at a daily or even higher frequency. This is important as an increasing number of derivatives exchanges mark-to-market positions twice a day (e.g., EUREX). Third, our approach differs from the "concentration risk" collateral method, which is most typically applied at the individual firm level. For instance, the Chicago Mercantile Exchange's clearing house monitors concentrations by focusing on the proportion of open interest on a given contract that is controlled by a single clearing firm, and it assigns additional margin to reflect the incremental exposure due to concentration.

In terms of methodology, this paper is at the confluence of two streams of literature. First, we rely on modeling techniques for extreme dependence as in Longin and Solnik (2001), Ang and Chen (2002), Poon, Rockinger, and Tawn (2004), Patton (2008), and Christoffersen et al. (2010). While previous papers focus on stock or hedge fund returns, we show that tail dependence can also be very useful to jointly model clearing members' P&L on a derivatives exchange. Second, our analysis builds on the recent literature on systemic risk. Adrian and Brunnermeier (2009) introduce the *CoVaR* measure that is the VaR of the financial system conditional on the distress of a given financial institution. Then they estimate the ΔCoVaR(firm i) = CoVaR(system|firm i) - VaR(system) that captures the marginal contribution of a particular institution to the overall systemic risk. Related studies by Acharya et al. (2010) and Brownlees and Engle (2010) focus on the *Marginal Expected Shortfall* of a given bank, defined as the expected loss of a particular firm conditional on the overall banking sector being in distress. Similar to these papers, we measure, and attempt to internalize, the potentially negative externalities of having interconnected market participants. Although in the same spirit, we use a totally different methodology and focus on margin requirements and the risk that correlated positions pose to the clearinghouse.

The outline of the paper is the following. In Section I, we show how to estimate tail dependence among clearing firm losses. In Section II, we formally describe our methodology to set collateral as a function of tail dependence. We compare the performance of our method to the standard margining system using simulations in Section III. Section IV summarizes and concludes our paper.

I. TAIL DEPENDENCE

In derivatives markets, margins serve as performance bonds to guard against default. In our work, the *performance bond* $B_{i,t}$ represents the margin requirement imposed by the clearing house on clearing firm i at the end of day t, for $i = 1, ..., \boldsymbol{N}$. This performance bond depends on the outstanding trading positions of the clearing firm. The *variation margin* $V_{i,t}$ represents the aggregate mark-to-market profit or loss of clearing firm i on day t. The *relative variation margin* $R_{i,t}$ is defined as:

$$R_{i,t} = \frac{V_{i,t}}{B_{i,t-1}} \tag{1}$$

Clearing firm i is in financial distress at time t if $R_{i,t} < -1$, or equivalently if $B_{i,t-1} + V_{i,t} < 0$, since in this case the trading loss exceeds posted collateral. In such a situation, the clearing firm may decide to default, which would generate a shortfall in the system that needs to be covered by the clearing house.

By definition, tail dependence measures the probability of two random variables having simultaneous extreme events in the same direction. We define the coefficients of upper and lower tail dependence to quantify the comovement in revenues across clearing firms in extreme market conditions. In our context, the tail dependence structure captures the degree of diversification across clearing firms and the likelihood of having simultaneous financial distress across several clearing firms. The coefficient of *upper tail dependence* of the relative variation margins of clearing firms i and j at time t is defined as:

$$\tau^{U}_{i,j} = \lim_{\alpha \to 1} Pr[R_i \geq F_i^{-1}(\alpha) \mid R_j \geq F_j^{-1}(\alpha)] = \lim_{\alpha \to 1} Pr[R_j \geq F_j^{-1}(\alpha) \mid R_i \geq F_i^{-1}(\alpha)] \tag{2}$$

where $F_i(R_i)$ denotes the marginal cumulative distribution function of R_i for $i = 1, ..., N$, and $a \in (0, 1)$ represents the marginal cumulative distribution level. Likewise, the coefficient of *lower tail dependence* of the relative variation margins of clearing firms i and j at time t is defined as:

$$\tau^{L}_{i,j} = \lim_{\alpha \to 0} Pr[R_i \leq F_i^{-1}(\alpha) \mid R_j \leq F_j^{-1}(\alpha)] = \lim_{\alpha \to 0} Pr[R_j \leq F_j^{-1}(\alpha) \mid [R_i \leq F_i^{-1}(\alpha)] \tag{3}$$

Because we are primarily concerned with shortfall in the clearing system, we focus on the lower tail and simplify the notation as follows: $\tau_{i,j} = \tau^{L}_{i,j}$.

We model trading revenue dependence across clearing firms by using a bivariate copula (Patton 2009). A copula is a function that links together marginal probability distribution functions, say $F_i(R_i)$ and $F_j(R_j)$, to form a multivariate probability distribution function, in this case $F(R_i,R_j)$. According to Sklar's Theorem (Sklar 1959), if the marginal distributions are continuous, there exists a unique copula function C such that:

$$F(R_i, R_j) = C(F_i(R_i),\ F_j(R_j)) \tag{4}$$

Several features of copulas are useful in our context. First, marginal distributions do not need to be similar to each other. Second, the choice of the copula is not constrained by the choice of the marginal distributions. Third, copulas can be used with N marginal distributions. Fourth, the use of copula functions enables us to model the tails of the marginal distributions and tail dependence separately. This last point is very important in our case because in a multivariate setting, the likelihood of an extreme event can increase either because of fatter tails in the marginal distributions or because of fatter tails in the joint distribution function.

A natural candidate that allows us to incorporate tail dependence is the Student t-copula. Let t_v be the univariate Student t probability distribution function with v degrees of freedom. Then, for continuous marginal distributions, $F_i(R_i)$, the bivariate Student t-copula, $T_{\rho,v}$, is defined as:

$$T_{\rho,v}\left(F_i(R_i), F_j(R_j)\right) = t_{\rho,v}(R_i, R_j) \tag{5}$$

where $t_{\rho,v}$ is the bivariate distribution corresponding to t_v and $\rho \in [-1,1]$ is the correlation coefficient between R_i and R_j.

A Student t-copula corresponds to the dependence structure implied by a multivariate Student t distribution. It is fully defined by the correlation of the implicit variables, ρ, and the degrees of freedom, v. The degrees of freedom define the probability mass assigned to the extreme co-movements of the relative variation margins (both positive and negative). In addition, this copula assigns a higher probability to joint extreme events, relative to the Gaussian copula, the lower the degrees of freedom, because a Student t copula with $v_t \rightarrow \infty$ corresponds to a Gaussian copula.

Student t-copulas allow us to readily obtain an estimate of the coefficient of lower tail dependence based on the correlation coefficient and the degrees of freedom (Cherubini, Luciano, and Vecchiato 2004):

$$\tau_{i,j} = 2t_{v+1}\left(-\sqrt{v+1}\sqrt{\frac{1-\rho}{1+\rho}}\right) \tag{6}$$

As can be seen from this equation, two parameters, the correlation coefficient and the degrees of freedom, fully describe the dependence structure of trading revenues. Intuitively, larger correlations and lower degrees of freedom lead to higher tail dependence.

We implement a two-stage semiparametric approach to estimate the pairwise copulas across all clearing firms. The first stage consists of estimating the empirical marginal distribution of the trading revenues of each clearing firm. The second stage consists of estimating the t-copula parameters, ρ and v, for every pair of clearing members through maximum likelihood (Genest, Ghoudi, and Rivest 1995).

II. COLLATERAL

In this section, we propose a new way of setting margin requirements for clearing firms. Our approach accounts for both tail risk and tail dependence structure across clearing firms. We consider a derivatives exchange with N clearing firms and D derivatives contracts (futures and options) written on U underlying assets. Let $w_{i,t}$ be the number of contracts in the derivatives portfolio of clearing firm i at the end of day t:

$$w_{i,t} = \begin{bmatrix} w_{i,1,t} \\ \vdots \\ w_{i,D,t} \end{bmatrix} \tag{7}$$

We consider two ways of computing the margin requirement of a clearing firm, which we present in turn below.

A. Standard Collateral Requirement

The standard collateral requirement is applied on a firm by firm basis, without regard to correlations across firms. As in the SPAN system utilized by the CME, we consider a series of S scenarios based on potential one-day ahead changes in the value (ΔX) and volatility ($\Delta\sigma_X$) of the underlying assets, as well as in the time to expiration of the derivatives products. For each of the S scenarios, we revaluate the portfolio (i.e., we “mark-to-model” its positions) and compute the associated hypothetical P&L or variation margin on the portfolio:

$$\tilde{V}_{i,t+1} = \begin{bmatrix} \tilde{V}^1_{i,t+1} \\ \vdots \\ \tilde{V}^S_{i,t+1} \end{bmatrix} \tag{8}$$

The *standard collateral requirement, B*, corresponds to the q% quantile of all simulated P&L across all considered scenarios:

$$Pr[\tilde{V}^s_{i,t+1} \leq -B_{i,t}] = q \tag{9}$$

with $s = 1,\ldots, S$. Thus, B accounts for the potential financial distress of a particular clearing firm, but it ignores its interdependence with other clearing members. In this standard case then, the total collateral collected by the clearing house at time t from all clearing firms is:

$$B_t = \sum_{i=1}^{N} B_{i,t} \tag{10}$$

B. Tail-Dependent Collateral Requirement

The tail-dependent collateral requirement is based not only on the magnitude of simulated losses (as in the standard collateral requirement) but also on the dependence structure across clearing firms' simulated losses. Our objective is to increase the collateral requirement for each individual firm by an amount proportional to its degree of dependence with other firms, with the increased collateral matching the incremental risk presented to the clearinghouse from potentially correlated losses among clearing members. Consider the portfolios of derivatives contracts of two clearing firms at the end of a given day:

$$w_{i,t} = \begin{bmatrix} w_{i,1,t} \\ \vdots \\ w_{i,D,t} \end{bmatrix} \qquad w_{j,t} = \begin{bmatrix} w_{j,1,t} \\ \vdots \\ w_{j,D,t} \end{bmatrix} \tag{11}$$

For each clearing firm, we compute the variation margins generated by the S scenarios described in the previous section:

$$\tilde{V}_{i,t+1} = \begin{bmatrix} \tilde{V}^1_{i,t+1} \\ \vdots \\ \tilde{V}^S_{i,t+1} \end{bmatrix} \qquad \tilde{V}_{j,t+1} = \begin{bmatrix} \tilde{V}^1_{j,t+1} \\ \vdots \\ \tilde{V}^S_{j,t+1} \end{bmatrix} \tag{12}$$

From Equation (12), we compute $B_{i,t}$ and $B_{j,t}$ as in equation (9). The tail dependence between the clearing firms' simulated relative variation margins is given by:

$$\tilde{\tau}_{i,j,t} = \lim_{\alpha \to 0} Pr\left[\tilde{R}_{i,t+1} \leq F^{-1}_{i,t+1}(\alpha) \middle| \tilde{R}_{j,t+1} \leq F^{-1}_{j,t+1}(\alpha)\right] \tag{13}$$

where $\tilde{R}_{i,t+1} = \tilde{V}_{i,t+1} / B_{i,t}$. With N clearing firms, we end up with $N(N-1)/2$ tail dependence coefficients, which can be presented in a lower diagonal matrix:

$$\begin{bmatrix} \tilde{\tau}_{2,1} & & & \\ \tilde{\tau}_{3,1} & \tilde{\tau}_{3,2} & & \\ \vdots & & \ddots & \\ \tilde{\tau}_{N,1} & \tilde{\tau}_{N,2} & \cdots & \tilde{\tau}_{N,N-1} \end{bmatrix}$$

For each clearing firm we conservatively set its collateral requirement as a function of the highest coefficient of tail dependence with respect to all other clearing firms:[3]

$$\tilde{\tau}_{i,t} = max\left\{\tilde{\tau}_{i,j,t}\right\}_{j=1,j\neq i}^{N} \tag{14}$$

$$B_{i,t}^{*} = B_{i,t} \times e^{max\left\{\gamma(\tilde{\tau}_{i,t}-\underline{\tau});0\right\}} \tag{15}$$

where γ is the tail-dependence aversion coefficient and $\underline{\tau}$ is a threshold tail dependence coefficient below which the collateral is not affected, that is: $B_{i,t}^{*} = B_{i,t}$ if $\tilde{\tau}_{i,t} \leq \underline{\tau}$. Thus, the total collateral collected by the clearing house becomes:

$$B_t^{*} = \sum_{i=1}^{N} B_{i,t}^{*} \tag{16}$$

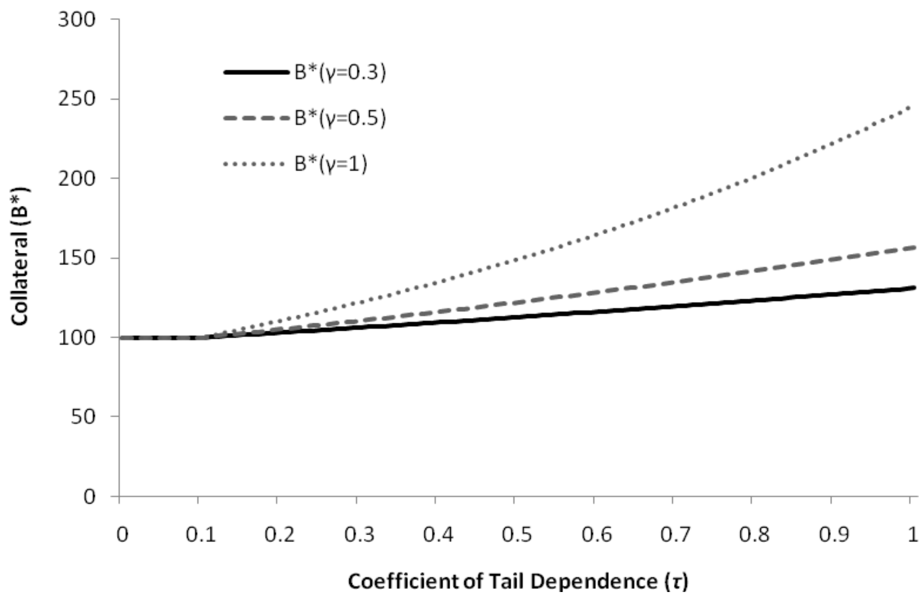
Notice that in the degenerative case where $\gamma = 0$ or if $\tilde{\tau}_{i,t} \leq \underline{\tau}$, we get the standard collateral requirement B. Thus, the standard collateral requirement (i.e., the SPAN system) is a special case of the tail-dependent collateral requirement. In other words, our approach can be seen as a generalized SPAN system. An implication of this result is that $B_t^{*} \geq B_t$. As an illustration, we plot in Figure 1 the level of tail-dependent collateral for different coefficients of tail-dependence aversion ($\gamma = 0.3, 0.5, 1$), $\underline{\tau} = 0.10$, $B = 100$, and for a tail parameter ranging between 0 and 1. Notice that no additional collateral is required for low coefficients of tail dependence. The required collateral increases with higher tail-dependence aversion, a choice variable for the clearing house, and with higher tail dependence, a parameter that can be estimated from simulated trading revenues.

III. CONTROLLED EXPERIMENT

To demonstrate the difference between the standard and tail-dependent collateral requirements, we consider a derivatives exchange with N clearing firms and two call options written on different underlying assets. Four clearing firms are assumed to be systemically important ($n = 4$) due to their size, so we focus on their margin requirements. Panel A of Table 1 displays the trading positions of these systemically important members in three different states: (1) low tail dependence, (2) moderate tail dependence, and (3) high tail dependence. The first state is obtained by selecting orthogonal trading positions across the systemically important firms. For the remaining states, the level of tail dependence is gradually increased by allowing the second firm to hold a position that progressively resembles that of the first. Notice, however, that the positions of the first, third, and fourth clearing firms remain constant across states. In addition, non-systemically important clearing firms

3. As a nested case, the standard collateral requirement case implies zero tail dependence.

Figure 1. Tail-Dependent Collateral.



Notes: This figure presents the level of tail-dependent collateral B^* as a function of the coefficient of tail dependence τ. The standard collateral requirement B is assumed to be equal to 100 and the threshold tail dependence coefficient $\underline{\tau}$ (below which collateral is not affected) to 0.10. The tail-dependence aversion coefficient γ of the clearing house varies between 0.3 and 1.

are assumed to clear the market in every state. Thus, each option contract is always in zero-net supply.

To simulate the variation margins for each clearing firm ($\widetilde{V}_{i,t+1}$), we define S scenarios that combine potential one-day changes in the value of the underlying assets, ΔX_1 and ΔX_2, with changes in volatility, $\Delta\sigma_{X1}$ and $\Delta\sigma_{X2}$. For each scenario, we mark-to-model the positions using the Black-Scholes model and generate a hypothetical change in the value of the portfolio held by each clearing firm. We then compute the coefficients of tail dependence between the simulated relative variation margins as described in equation (13). Panel B of Table 1 shows the estimated coefficients of tail dependence, and Table 2 shows the parameter values used for this controlled experiment.

Panel C of Table 1 compares three ways of computing collateral. The first two are the standard margin requirement (B) and the tail-dependent margin requirement (B^*) discussed earlier.[4] The third collateral system aims at being budget-neutral, and it provides a better benchmark against which to compare the tail-dependent margining system because it collects the same aggregate collateral. This *budget-neutral* margin requirement is defined as:

4. See equation (9) for the definition of the standard margin requirement (B) and equation (15) for the definition of the tail-dependent margin requirement (B^*).

Table 1. Controlled Experiment.

	Low Tail Dependence				Moderate Tail Dependence				High Tail Dependence			
	1	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Panel A: Trading Positions												
$d = 1$	100	30	-50	-100	100	125	-50	-100	100	95	-50	-100
$d = 2$	100	-170	150	-100	100	75	150	-100	100	105	150	-100
Panel B: Tail Dependence Coefficients												
$\tilde{\tau}_{2,j}$	.000	.	.	.	.247	.	.	.	.908	.	.	.
$\tilde{\tau}_{3,j}$	.000	.000	.	.	.000	.000	.	.	.000	.000	.	.
$\tilde{\tau}_{4,j}$	.000	.000	.000	.	.000	.000	.000	.	.000	.000	.000	.
$\tilde{\tau}_i$	.000	.000	.000	.000	.247	.247	.000	.000	.908	.908	.000	.000

Table 1, continued. Controlled Experiment.

	Low Tail Dependence				Moderate Tail Dependence				High Tail Dependence			
	1	2	3	4	1	2	3	4	1	2	3	4
Panel C: Margins												
B_i	3,849	6,228	4,310	5,319	3,849	3,918	4,310	5,319	3,849	3,851	4,310	5,319
B_i^*	3,849	6,228	4,310	5,319	4,022	4,094	4,310	5,319	4,905	4,908	4,310	5,319
B_i^0	3,849	6,228	4,310	5,319	3,936	4,005	4,397	5,406	4,377	4,380	4,839	5,847
p_i	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
p_i^*	.050	.050	.050	.050	.041	.038	.050	.050	.007	.007	.050	.050
p_i^0	.050	.050	.050	.050	.045	.044	.046	.046	.022	.022	.026	.033

Notes: Panel A presents the trading positions of the four systemically important clearing firms in two option contracts ($d = 1,2$) when tail dependence is low, moderate, and high. Panel B displays the tail dependence coefficients among pairs of clearing firms ($\tilde{\tau}_{i,j}$) and the highest coefficient of tail dependence across all pairs ($\tilde{\tau}_i$). Panel C show the standard margins (B_i), the tail dependent margins (B_i^*), and the budget-neutral margins (B_i^0). When computing tail dependent margins, we use a tail-dependence aversion coefficient γ of 0.3 and a threshold tail dependence coefficient $\underline{\tau}$ of 0.1. Finally, the p_i variables denote the probability of a clearing firm being in financial distress: $p_i = \Pr[V_{i,t+1} \leq -B_{i,t}]$, $p_i^* = \Pr[V_{i,t+1} \leq -B_{i,t}^*]$, and $p_i^0 = \Pr[V_{i,t+1} \leq -B_{i,t}^0]$.

Table 2. Controlled Experiment Parameters.

Parameter	Value
A. Market and Clearing Members	
Number of derivatives securities (D)	2
Number of underlying assets (U)	2
Number of systemically important clearing members (n)	4
B. Underlying Assets	
Value of underlying asset 1 at $t = 0$	$100
Value of underlying asset 2 at $t = 0$	$100
C. Derivatives Securities	
Strike price of option contract 1	$100
Strike price of option contract 2	$100
Time to maturity of option contract 1	1 year
Time to maturity of option contract 2	1 year
D. Margining Systems	
Variation range in the value of the underlying assets	± 50%
Variation range in the volatility of the underlying assets' returns	± 50%
Number of scenarios for the value of the underlying asset and its volatility (S)	10,000
Quantile for the standard collateral system (q)	5%
Tail-dependence aversion coefficient (γ)	0.3
Threshold tail-dependence coefficient ($\underline{\tau}$)	0.1

$$B_{i,t}^{0} = B_{i,t} + \frac{B_{t}^{*} - B_{t}}{n} \text{ for } i = 1,\ldots,n \tag{17}$$

where the budget-neutral condition is:

$$\sum_{i=1}^{n} B_{i,t}^{0} = \sum_{i=1}^{n} B_{i,t}^{*} \tag{18}$$

and from equation (15) it follows that $B_{i,t}^{0} = B_{i,t} = B_{i,t}^{*}$ when $\tilde{\tau}_{i,t} \leq \underline{\tau}$ or $\gamma = 0$.

The results presented in Panel C of Table 1 show that the three margining systems are equivalent in the low-dependence state and that they diverge as the level of tail dependence increases to 0.247 in the moderate-dependence state, and to 0.908 in the high-dependence state. The equivalence across margining systems in the low-dependence state arises because the tail dependence coefficients are virtually zero; thus, $\tilde{\tau}_{i,t} \leq \underline{\tau}$ and $B_{i,t}^{*} = B_{i,t}$ for all clearing firms. In other words, when default risk is well-diversified among clearing firms, the tail-dependent margining system converges to the standard system. On the other hand, in the moderate and high-dependence states, the tail-dependent margin requirement for clearing firms 1 and 2 increases due to their progressively homogeneous trading positions. This homogeneity is captured by the higher coefficient of tail dependence that is incorporated into B^*.

Notice, however, that the standard and tail-dependent margin requirements of firms 3 and 4 remain unchanged across states as their positions stay orthogonal relative to those of the other members. This is not true for the budget-neutral case. The budget-neutral margin requirement increases for all members in the moderate and high-dependence states. This situation arises because the additional collateral that would be collected under the tail-dependent margining system is now collected across all systemically important clearing firms. As a consequence, the budget neutral collateral requirements of firms 3 and 4 increase in the moderate and high dependence state due to the increased tail dependence between firms 1 and 2.

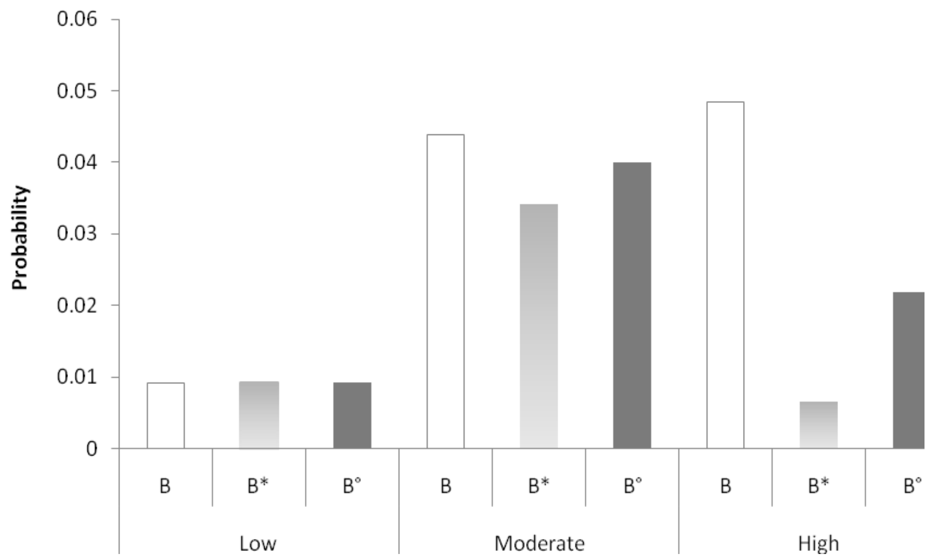
In order to assess the appropriateness of each margining system, we now turn our attention to their relative performance. We simulate changes in the value of the call options by randomly selecting one of the S scenarios. For each margining system, we compute the probability of financial distress across clearing firms, the probability of joint financial distress, and the magnitude of the average margin shortfall given joint financial distress. The bottom part of Panel C in Table 1 shows the probability of financial distress (i.e., the probability that $B_{i,t-1} + V_{i,t} < 0$) across clearing firms. Since the quantile for the standard margining system, q, was set to 5% in the simulation (see Table 2), the standard system has a distress probability of 5% in all scenarios by construction.

Similarly, in the low-dependence state, when $B_{i,t} = B^*_{i,t} = B^0_{i,t}$ for all clearing firms, the probability of financial distress is 5% across margining systems. In the moderate and high-dependence states, however, the distress probability is lower for firms 1 and 2 under the tail-dependent system and lower for all firms under the budget neutral system because more collateral is required relative to the standard case.

At first glance, this result would suggest that the budget neutral system performs better than the alternatives because it reduces the unconditional probability of financial distress across clearing firms. However, Figure 2 shows that the tail-dependent margining system actually provides a better allocation of margin requirements. More specifically, the figure shows that the probability of joint financial distress (i.e., the probability of one or more clearing firms jointly experiencing a loss in excess of their posted margin) is lower under the tail-dependent margining system, particularly when tail dependence is high.

Notice that the probability of joint financial distress increases monotonically with tail dependence under the standard collateral system. Differently, for the tail-dependent system, this probability first increases in the moderate-dependence state and then decreases in the high-dependence state. This result arises due to the value of the tail-dependence aversion coefficient, $\gamma = 0.3$, and the value of the threshold tail dependence coefficient, $\underline{\tau} = 0.1$ (see Table 2), which translates into a slight increase in the required margin for firms 1 and 2 (an additional \$173 and \$176,

Figure 2. Probability of Joint Financial Distress



Notes: This figure presents, for each margining system, the likelihood of several clearing firms jointly being in financial distress, i.e., $B_{i,t-1} + V_{i,t} < 0$, with different levels of tail dependence between clearing firms (low, moderate, and high). The three systems are the standard (B), tail-dependent margin requirement (B^*), and budget-neutral (B^0) margin requirement systems. The results are based on 1,000,000 simulations of the actual changes in the underlying asset prices.

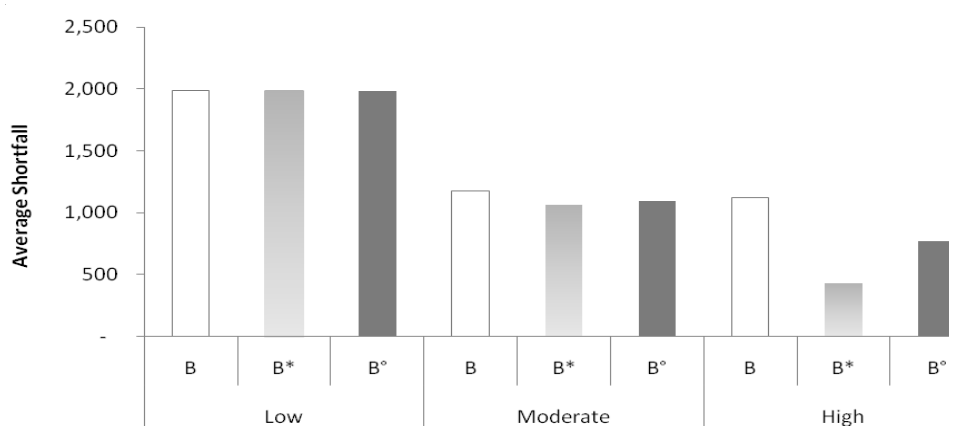
respectively) in the moderate tail-dependence state, and a significantly larger increase (an additional \$1,056 and \$1,057, respectively) in the high tail-dependence state. Similar results can be observed in the budget neutral system for the same reasons. A monotonic decrease of the probability of joint financial distress could be obtained for the tail-dependent collateral system if a higher value of γ or a $\underline{\tau}$ of 0 is selected.

Finally, Figure 3 shows that the average shortfall ($B_{i,t-1} + V_{i,t}$), given financial distress, is lower under the tail-dependent margining system in the moderate and high-dependence states. Therefore, we can conclude that the tail-dependent margining system is superior to the other systems because it provides a better allocation of margin requirements. This allocation depends on the composition and homogeneity of the trading positions of the clearing members and it provides better protection against joint negative outcomes.

IV. CONCLUDING REMARKS

In this paper, we present a novel approach to compute margins for a portfolio of derivatives securities. The innovative feature of this method is to account not only for the riskiness of the trading positions of an individual market participant but also for the interdependence between this participant's trading positions and *other* participants' trading positions. Our method is a simulation-based technique that accounts for extreme tail dependence among potential trading losses. Accounting

Figure 3. Average Shortfall Given Joint Financial Distress.



Notes: This figure presents, for each margining system, the average shortfall ($B_{i,t-1} + V_{i,t}$) given joint financial distress with different levels of tail dependence between clearing firms (low, moderate, and high). The three systems are the standard (B), tail-dependent margin requirement (B^*), and budget-neutral (B^0) margin requirement systems. The results are based on 1,000,000 simulations of the actual changes in the underlying asset prices.

for interconnections among clearing firms in a derivatives exchange is shown to lower the probability of several clearing members being simultaneously in financial distress (i.e., when losses exceed posted collateral), as well as the magnitude of the margin shortfall given joint financial distress, which decreases systemic risk concerns.

While our simulation analysis focuses on margins for option positions, our method can be applied to any listed derivatives contract such as futures, swaps, or exchange-traded credit derivatives. Furthermore, it is important to realize that our approach should by no means be limited to derivatives exchanges and can also be used to set collateral in any financial network. For instance, our method could be used to set collateral requirements for OTC positions as well.

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